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Dirac's Electron

Dirac's electron is an abstract product of his mathematical enterprise. The electron I have in mind is a sphere filled by electric charge. It is an electron which Dirac, the authority on electrons, was able to dismiss in 1938* with the words:

> The Lorentz model of the electron as a small sphere charged with electricity, possessing mass on account of the energy of the electric field around it, has proved very valuable in accounting for the motion and radiation of electrons in a certain domain of problems, in which the electromagnetic field does not vary too rapidly and the accelerations are not too great. Beyond this domain it will not go unless supplemented by further assumptions about the forces that hold the charge of an electron together. No natural way of introducing such further assumptions has been discovered and it seems that the Lorentz model has reached the limit of its usefulness and must be abandoned before we can make further progress.

Dirac's criticism is the problem of the forces that hold the electron together. This is an extremely basic question in physics. It defies explanation, until you have seen the very simplicity of the answer. Force and pressure are not primary phenomena. Force does not act instantaneously between electric charge. Force occurs only when energy changes and energy changes only when it can. The primordial parameters are taken here as space and electric charge. Given a volume of space occupied by electric charge, we can say that the mutual repulsion 'forces' in the charge will cause it to adopt spherical form. Yet, the volume and the charge jointly determine the energy and energy determines the form. To explain this, imagine a definite volume of space bounded by fixed walls, as depicted in Fig. 2. Within these walls we presume there to be a medium filling the space except

* 'Classical Theory of Radiating Electrons', P. A. M. Dirac, *Proc. Roy. Soc.*, 167, p. 148, 1938.

for a sphere occupied by a definite quantity of electric charge. Instead of regarding the charge as self-repulsive, assume that it develops energy in the surrounding field. If this field energy can

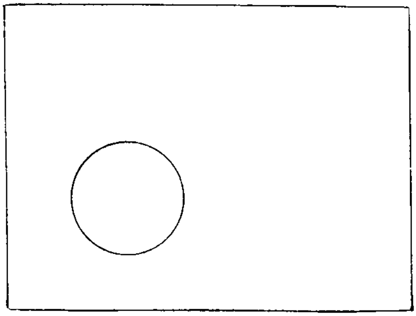

Fig. 2

change, then the charge can redeploy. Reduction of the field energy by transfer of energy to some other form can result in the charge expanding, as we understand by the notion of a repulsive force action. But there has to be reason for energy change. We take it that the charge has the character of preserving itself, of moving to conserve its energy if subject to extraneous influence. This is the subject of the next chapter. However, here we make the point that the charge cannot expand unless the space it occupies, the sphere bounding it, expands first. If it did expand without such assistance, its energy would disperse and there would be no discrete electric charges in the whole of our universe. We must take note of the fact that electrons and many other elementary charged particles are stable. Force arises only when energy changes and this is only when motion can occur to permit the change.

Now, if the walls of the system depicted in Fig. 2 move outwards, it is a different story. The particle of charge will become unstable. It will expand and release energy. Of course, in Nature, the imaginary space is full of charges. They can interact without change of volume. They occupy the same volume whether they are close together or far apart. Hence their interaction energy can change to develop forces between the charges which we measure and from which we reduce Coulomb's law. If the walls of space expand, first one and then another of the charges will expand by a statistical process. As soon as the first one starts to expand it will act to fill the space by displacing the filling medium and so restrain other particles of charge from expanding at the same instant. We are here envisaging a process of expansion of the universe by which more space is constantly created as the plenum of electric charge constantly, but statistically with its transiently stable states, expands to keep voids from forming.

Returning to Dirac, we can say that Nature keeps the electron charge tightly together for our purposes and, while it is useful to have an explanation even coupled with assumptions, the fact does remain that the Lorentz model of the electron can survive as a viable idea. Quoting further from Dirac's paper:

> One of the most attractive ideas in the Lorentz model of the electron, the idea that all mass is of electromagnetic origin, appears at the present time to be wrong, for two separate reasons. First, the discovery of the neutron has provided us with a form of mass which it is very hard to believe could be of electromagnetic nature. Secondly, we have the theory of the positron—a theory in agreement with experiment so far as is known—in which positive and negative values for the mass of an electron play symmetrical roles. This cannot be fitted in with the electromagnetic idea of mass, which insists on all mass being positive, even in abstract theory.

Neutrons are believed by some authorities to be electrically neutral aggregations of discrete electric charges of opposite polarities. This is the basis of what has come to be called quark theory, but we do not have to believe all about quarks to accept this aggregation idea. Furthermore, as we observed in Chapter 9, the current belief was that Earnshaw's theorem denied the possibility of stable aggregations of this kind and this belief is ill-founded. Also, since Dirac wrote the above words it has been discovered that a neutron can be diffracted by a magnetic field and this does suggest that it has an electrical form. Certainly, it is not by any means reasonable to argue today that the mass of the neutron is not characteristic of its electrical nature. Dirac's second point, that the theory of the positron implies an electron of negative mass, is hardly pertinent. It merely sets his theory against the logical and physically founded concepts of the ever-positive mass effect of electromagnetic field energy. It is like saying that his theory conflicts with the other one and is correct solely for this reason.

Then Dirac, whose object is covered by the title of his paper, 'Classical Theory of Radiating Electrons' goes on to say how it is desirable to assume a point model for the electron to avoid the unnecessary complication of not having the field equations he uses 'holding all the way up to the electron's centre'.

At this stage we must pause for reflection. We are examining Dirac's thoughts on the question of energy radiation by the accelerated electron. Dirac wants to use a point charge electron whose mathematical portrayal invokes field equations applicable throughout space. That is, boundary problems are to be put aside. The reader, if he is tuned into the author's viewpoint, may wish to retain the electron as a sphere of electric charge, if only because it is easier to imagine a finite object than a mere point surrounded by mathematical equations. These differences are important if we are to end up with something meaningful.

Dirac then runs into the obvious problem that the energy of the electron would become infinite if Maxwell's theory is to hold. The self-energy of an electric charge is inversely proportional to the spacing between its charge elements. The spacing is zero if the charge is concentrated at a point. So Dirac declares that he does not want to reject Maxwell's theory and that he will try to overcome the difficulty by mathematics. He writes:

> Our aim will be not so much to get a model of the electron as to get a simple scheme of equations which can be used to calculate all the results that can be obtained from experiment.

This seems an appropriate objective, but we are looking at a paper about energy radiation by electrons and it is a fact that no one has ever, even to this day, measured experimentally energy radiated by discrete electrons. Energy transfer associated with radiation, which in its turn is associated with the excitation of electrons in test apparatus, has been observed, but when Dirac speaks of calculation it is not merely energy transfer which has to result from his equations. It is quantitative data of energy transfer which permits verification by experiment and theory on this is an idle pursuit if we have no way of relating this to the specific number of electrons present and contributing.*

Dirac then brings us to the following statement:

> A great deal of work has been done in the past in examining the general implications of Maxwell's theory, but it was nearly all done before the discovery of quantum mechanics in 1925, when people gave all their attention to the question of how an electron could remain in an atomic orbit without radiating—a question we now know can be answered only by going outside classical theory—and were thus not interested in simply looking for the most natural interpretation their equations would allow.

This, indeed, is a statement which evokes comment. If everyone faced the question of how an electron could remain in an atomic orbit without radiating, why is it that this was not taken as the clue to one of the most fundamental questions in physics, the question about the very nature of mass? The mass of the electron could well be that property it exhibits in moving to conserve its intrinsic electric field energy and so its charge. Why go outside classical theory to couple with quantum theory an over-riding restraint on energy radiation? Why bother interpreting equations? If an electron in an atomic orbit does not radiate energy then an electron need not radiate energy whether accelerating by moving steadily in a circular orbit or accelerating in a straight line. This is the simple interpretation and, if equations indicate otherwise, we must question whether they are built upon erroneous assumptions.

However, Dirac did not do this. He was writing about the radiation of energy by electrons according to classical theory and if atomic electrons did not radiate energy quantum radiation assumptions were too easy a way of avoiding the problem. Dirac wanted to stay with the mathematical equations and draw meaning from them. He even added strength to the classical theory by using relativistic principles to derive the usual expression for energy radiation according to Lorentz's theory, and he wrote:

* See discussions of Cerenkov radiation in Chapter 12.

Whereas these equations, as derived from the Lorentz theory, are only approximate, we now see that there is good reason for believing them to be exact, within the limits of classical theory.

Then, on the next page of his paper:

As an interesting special case, let us suppose there is no incident field, so that we have the equations of motion . . .* In general the electron will not now be moving with constant velocity, as it would according to ordinary ideas, since we may suppose it to be started off with a non-zero acceleration and it cannot then suddenly lose its acceleration.

This is a fantastic result to anyone accustomed to Newtonian mechanics. Dirac realizes this when he then writes:

To study the rather unexpected results of the preceding section more closely. . . . It would appear that we have a contradiction with elementary ideas of causality. The electron seems to know about the pulse before it arrives and to get up an acceleration (as the equations of motion allow it to do), just sufficient to balance the effect of the pulse when it does arrive.

Surely this just cannot be believed. Dirac was basing his analysis upon acceptance of an idea presented by Schott in the *Philosophical Magazine* in 1915.† Schott had analysed the problem of the incident electric field and wrote:

This equation shows that the whole of the work done by the external field is converted into kinetic energy of the electron just as if there had been no radiation at all. None of it is radiated. . . . Thus we see that the energy radiated by the electron is derived entirely from its acceleration energy.

Schott's idea was to provide the electron with an energy component he called 'acceleration energy' of which he said:

Its existence is a direct consequence of a mechanical theory of the aether.

So convinced were the physicists involved in these studies that the electrons must radiate energy if accelerated that they had to look to the physical force exerted by a mechanical, as opposed to

* These equations contained acceleration terms even though Dirac specifies no incident field able to exert force on the electron.

† Vol. 29, pp. 49–62.

an electrical, aether to call into account the sources of the energy radiated.

So, here was Dirac in 1938, adopting the notion of an acceleration energy necessitating aether able to exert mechanically the forces needed to feed the energy being radiated and, apparently, missing the obvious fact that the easy way out of all the difficulties is to see that the electron does not radiate energy at all. Dirac was addressing a problem which did not exist, and now see where he was guided by his conclusions:

> The behaviour of our electron can be interpreted in a natural way, however, if we suppose the electron to have a finite size. There is then no need for the pulse to reach the centre of the electron before it starts to accelerate.

Yet he started his paper by saying that the electron should be deemed to be a point charge! Then he wrote:

> Mathematically, the electron has no sharp boundary and must be considered as extending to infinity.

This is puzzling. It depends whether we have charge or energy in mind. If we have stayed with the model of the electron as a sphere of charge, we can see a finite electron, meaning the charge, and also see a field extending to infinity.

Finally, Dirac concluded:

> In this way a signal can be sent from A to B faster than light. This is a fundamental departure from the ordinary ideas of Relativity and is to be interpreted by saying that it is possible for a signal to be transmitted faster than light through the interior of the electron. The finite size of the electron now reappears in a new sense, the interior of the electron being a region of failure, not of the field equations of electromagnetic theory, but of some elementary properties of space-time.

Space–time has failed. What does this mean? How can space–time fail? If the space–time according to Relativity fails, then Relativity fails. But how can anyone accept the argument presented here by Dirac? It is submitted that the question of the radiation of energy by an electron was clarified by Dirac's paper to the extent that the paper demonstrated the impossible situation into which mathematical formalism can lead the physicist. Modern physical theory has become abstract. The starting points of the original papers on the subject are mathematical, the treatment is mathematical and the conclusions are mathematical. In many instances there seems to be no relation whatsoever to the phenomena which make up the world of experimental physics. Dirac has been bold enough to translate his findings into language which can be interpreted in the context of a true understanding of Nature. He has revealed a maze in which so many physicists seem to be wandering, following one another, without having any clear direction in which to go. It is due time that this was realized. This realization is the key to further progress, as we see in the next part of this work.

Whereas Dirac, incidentally, declares that space–time fails within the electron but Maxwell's equations operate, Einstein, in his book *The Meaning of Relativity*, first published in 1922, writes:

We do know, indeed, that electricity consists of elementary particles (electrons, positive nuclei), but from a theoretical point of view we cannot comprehend this. We do not know the energy factors which determine the distribution of electricity in particles of definite size and charge, and all attempts have failed. If then we can build upon Maxwell's equations at all, the energy tensor of the electromagnetic field is known only outside the charged particles. It has been attempted to remedy this lack of knowledge by considering the charged particles as proper singularities. But in my opinion this means giving up a real understanding of the structure of matter. It seems to me much better to admit our present inability rather than to be satisfied by a solution that is only apparent.

It should be mentioned that Dirac himself wrote in *Scientific American* in May, 1963:

I might mention a third picture with which I have been dealing lately. It involves departing from the picture of the electron as a point and thinking of it as a kind of sphere with a finite size . . . the muon should be looked on as an excited electron. If the electron is a point, picturing how it can be excited becomes quite awkward.